THE MATHEMATICAL BACKGROUND OF
LOMONOSOV'S CONTRIBUTION

S. S. KUTATELADZE

ABSTRACT. This is a short overview of the influence of mathematicians and their ideas on the creative contribution of Mikhaĭlo Lomonosov on the occasion of the tercentenary of his birth.

Lomonosov is the Russian colossus of the epoch of scientific giants. Lomonosov was not a mathematician, but without mathematicians there would be no Lomonosov as the first and foremost Russian scientist at all.

Science in Russia had started with the foundation of the Academy of Sciences and Arts which then evolved into the Russian Academy of Sciences of these days. The turn of the sixteenth and seventeenth centuries is a signpost of the history of the mankind, the onset of the organized science. The time of the birth of scientific societies and academies accompanied the revolution in the natural sciences which rested upon the discovery of differential and integral calculus. The new language of mathematics brought about an opportunity to make impeccably precise predictions of future events.

To the patriotism of Peter the Great and the cosmopolitanism of Leibniz we owe the foundation of the Saint Petersburg Academy of Sciences as the center of Russian science. Peter and Leibniz stood at the cradle of Russian science in much the same way as Euler and Catherine I are the persons from whom we count the history of the national mathematical school in Russia. We must also acclaim the outstanding role of Leibniz who prepared for Peter a detailed plan of organizing academies in Russia (cp. [1]). Leibniz viewed Russia as a bridge for connecting Europe with China whose Confucianism would inoculate some necessary ethical principles for bringing moral health to Europe (cp. [2]). Peter wanted to see Leibniz as an active organizer of the Saint Petersburg Academy, he persuaded Leibniz in person and appointed Leibniz a *Justizrat* with a lavish salary. Elisabeth Charlotte d'Orléans who was present at the meeting of Peter with Leibniz wrote on December 10, 1712 (cp. [3]):

> Muscovy must be a savage place. Thus I find that Herr Leibniz is right in not wishing to go there. I am as it were charmed by the Tsar, when I see how much care he takes to improve his country.

It is worth observing that Peter visited the Royal Mint in London in 1698 during the so-called "Grand Embassy." At that time Newton was Warden of the Mint and we can hardly imagine that he ignored Peter's visits. Nevertheless there is no evidence that Peter met Newton. It is certain that Jacob Bruce, one of the closest associates of Peter, had discussions with Newton (cp. [4, p. 199]). In 1714, two years

---

after Peter made Leibniz a *Justizrat*, Aleksandr Menshikov applied for membership in the London Royal Society, which was an extraordinary and unforeseeable event. What is more mysterious, Menshikov's application was approved and he was notified of his new status by a letter from Newton himself (cp. [5, Ch. 16]).

The genius of Newton has revealed to the universe the mathematical laws of nature, disclosed to a mathematician a universal language for describing the ever-changing world. The genius of Leibniz has pointed out to the mankind the opportunities of mathematics as a reliable method of reasoning, the genuine logic of human knowledge. Leibnizian ideas of *mathesis universalis* and *calculemus* arose once and forever as a dream and instrument of science.

The influence of the ideas of Newton and Leibniz resulted in the the scientific outlook (e.g., [6, Ch. 2]). The revolt of the natural sciences at the turn of the seventeenth and eighteenth centuries was determined by the invention of differential and integral calculus. The competing ideas of the common mathematics of Newton and Leibniz determined all principal trails of thought of the intellectual search of the epoch. The contribution of Lomonosov exhibits a convincing example of the general trends. To grasp the scientific approaches of Lomonosov, to understand his creative revelations and naive delusions is impossible without deep analysis of and thorough comparison between the views of Newton and Leibniz.

The monads of Leibniz as well as the fluxions and fluents of Newton are products of the heroic epoch of the telescope and microscope. The independence of the discoveries of Leibniz and Newton is obvious, since their approaches, intellectual backgrounds, and intentions were radically different. Nevertheless, the groundless priority quarrel between Leibniz and Newton has become the behavioral pattern for many generations of scientists. Leibniz and Newton discovered the same formulas, part of which had already been known. Leibniz, as well as Newton, had his own priority in the invention of differential and integral calculus. Indeed, these scientists suggested the versions of mathematical analysis which were based on different grounds. Leibniz founded analysis on actual infinitesimals, erecting the tower of his perfect philosophical system known as monadology. The key of Newton was his method of "prime and ultimate ratios" which is rightfully associated with the modern limit theory.

The Leibnizian stationary vision of mathematical objects counterpoises the Newtonian dynamical perception of ever-changing variable quantities. The source of the ideas by Leibniz was the geometrical views of antiquity which he was enchanted with from his earliest infantry. The monad of Euclid is the mathematical tool of calculus, presenting a twin to the point, the atom of geometry. Mathematics of Euclid is the product of the human spirit. The monads of Leibniz, nurtured by his dream of *calculemus* are the universal instrument of creation whose understanding brings a man to the divine providence in creating the best of all possible worlds.

The point and the monad of the ancients are independent forms of reasoning, mental reflections of indivisible constituents of figures and numbers. Both ideas are tightly woven into the conception of universal atomism. The basic idea of the straight line has incorporated the understanding of its dualistic—discrete-continuous—nature from the very beginning of geometry. Leibniz ascribed the universal meaning to the ancient geometrical idea, discerning the divine providence that is incorporated in the idea.

Newton got acquaintance with Euclid only in his ripe years and so he travelled in his own way, perceiving universal motion as something done at the creation of the world that could thus never be reduced to any sum of states of rest. The perfectly precise characterization of Newton was done by Keynes in his talk [7] prepared to the tercentenary of Newton which had to be celebrated in 1942, but was postponed until 1946 because of the circumstances of wartime. Unfortunately, Keynes had passed away three months before the celebration and his lecture was delivered by his brother. Keynes wrote:

> Why do I call him a magician? Because he looked on the whole universe and all that is in it as a riddle, as a secret which could be read by applying pure thought to certain evidence, certain mystic clues which God had laid about the world to allow a sort of philosopher's treasure hunt to the esoteric brotherhood. He believed that these clues were to be found partly in the evidence of the heavens and in the constitution of elements (and that is what gives the false suggestion of his being an experimental natural philosopher), but also partly in certain papers and traditions handed down by the brethren in an unbroken chain back to the original cryptic revelation in Babylonia. He regarded the universe as a cryptogram set by the Almighty—just as he himself wrapt the discovery of the calculus in a cryptogram when he communicated with Leibniz. By pure thought, by concentration of mind, the riddle, he believed, would be revealed to the initiate."

If Newton was the last scientific magician, then Leibniz was the first mathematical dreamer.

The outlook of Leibniz, proliferating with his works, occupies a unique place in human culture. We can hardly find in the philosophical treatises of his predecessors and later thinkers something comparable with the phantasmagoric conceptions of monads, the special and stunning constructs of the world and mind which precede, comprise, and incorporate all the infinite advents of the eternity. It is worth emphasizing that mathematics was the true source of of the philosophical ideas of Leibniz. Suffice it to quote Child who translated into English and commented the early mathematical papers of Leibniz (cp. [8, Preface]):

> The main ideas of his philosophy are to be attributed to his mathematical work, and not *vice versa*.

Monadology [9, pp. 413–428] is usually dated as of 1714. This article was never published during Leibniz's life. Moreover, it is generally accepted that the very term "monad" had appeared in his writings since 1690 when he was already an established and prominent scholar.

The special attention to the origin of the term "monad" and the particular investigation into the date of its first appearance in the works by Leibniz are in fact the present-day products. There are now a few if any cultivated persons who never got acquaintance with the basics of planimetry and heard nothing of Euclid. However, no one has ever met the concept of "monad" on the school bench. Neither the contemporary translations of Euclid's *Elements* nor the popular school textbooks contain this seemingly exotic term. However, the concept of "monad" is fundamental not only for Euclidean geometry but also for the whole science of the Ancient Hellas.

By Definition I of Book VII of Euclid's *Elements* [10] a monad is "that by virtue of which each of the things that exist is called one." Euclid proceeds with Definition 2: "A number is a multitude composed of monads." Note that the present-day translations of the Euclid treatise substitute "unit" for "monad."

A contemporary reader can hardly understand why Sextus Empiricus, an outstanding scepticist of the second century, wrote when presenting the mathematical views of his predecessors as follows [11]: "Pythagoras said that the origin of the things that exist is a monad by virtue of which each of the things that exist is called one." And furthermore: "A point is structured as a monad; indeed, a monad is a certain origin of numbers and likewise a point is a certain origin of lines." Now some place is in order for the excerpt which can easily be misconceived as a citation from *Monadology*: "A whole as such is indivisible and a monad, since it is a monad, is not divisible. Or, if it splits into many pieces it becomes a union of many monads rather than a [simple] monad."

It is worth observing that the ancients sharply perceived an exceptional status of the start of counting. In order to count, one should firstly particularize the entities to count and only then to proceed with putting these entities into correspondence with some symbolic series of numerals. We begin counting with making "each of the things one." The especial role of the start of counting is reflected in the almost millennium-long dispute about whether or not the unit (read, monad) is a natural number. We feel today that it is excessive to distinguish the key role of the unit or monad which signifies the start of counting. However, this was not always so.

From the times of Euclid, all serious scientists knew about existence of the two basic concepts of mathematics: a point and a monad. By Definition 1 of Book 1 of Euclid's *Elements*: "A point is that which has no parts." Clearly this definition differs drastically from the definition of monad as that which makes one from many. The cornerstone of geometry is other than that of arithmetic. Without clear understanding of this circumstance it is impossible to comprehend the essence of the views of Leibniz. By the way, the modern set theory refers to "that which has no parts" as the empty set, the starting cardinal of the von Neumann universe. The present-day mathematics seems to have no concept that is vocalized as "that which many makes into one." We will return to the modern mathematical definition of monad shortly.

As a top mathematician of his epoch, Leibniz was in full command of Euclidean geometry. Therefore, rather bewildering is Item 1 of *Monadolody* where Leibniz gave the first idea of what his monad actually is:

> The Monad, of which we shall here speak, is nothing but a simple substance, which enters into compounds. By "simple" is meant "without parts."

This definition of monad as a "simple" substance without parts coincides with the Euclidean definition of point. At the same time the reference to compounds consisting of monads reminds us the structure of the definition of number which belongs to Euclid.

The synthesis of both primary definitions of Euclid in the Leibnizian monad is not accidental. We must always bear in mind that the seventeenth century is the epoch of microscope. It was already in the 1610s that microscopes were mass-produced in many European countries. From the 1660s Europe was enchanted by Antony van Leeuwenhoek's microscope.

Attempting to pursue the way of Leibniz's thought, we must always keep in mind that he was a mathematician by belief. From his earliest childhood, Leibniz dreamed of "some sort of calculus" that operates in the "alphabet of human thoughts" and possesses the same beauty, strength, and integrity as mathematics in solving arithmetical and geometrical problems. Leibniz devoted many articles to invention of this universal logical calculus. He remarked that his general methodological views are grounded on the "studying of the ways of analysis in mathematics to which I was subjected with such an ardency that I do not know whether there are many to be found today who invested much more toil into it than me."

The teacher of Lomonosov was Christian Wolff, an ardent propagator of the ideas of monadism and the mathematical method. Wolff was considered by his contemporaries as the second figure after Leibniz in the continental science. The first figure of Misty Albion was Newton. It is impossible to forget that the intellectual life of that epoch was heavily contaminated with the nasty controversy about priority between Newton and Leibniz. The deplorable consequence of the confrontation was the stagnation and isolation of the mathematical life of England. As regards the continent, the slight but perceptible neglect to the contribution by Newton led to dogmatization and canonization of the teaching of Leibniz which was often understood with distortions.

Wolff was an epigone rather than a follower of Leibniz. Tore Frängsmyr remarked in [12, p. 34]:

> Wolff's epistemology is quite simple. Human knowledge outside Christian revelation can be acquired in three ways: by experience (historical knowledge), by reason (mathematical knowledge), or by a combination of the two (philosophical knowledge). The last of these three methods is preferable; the other two have value only to the extent that they can be of use to philosophy.

The true disciples of Leibniz were Jacob, Jean, and Jacques Bernoulli as well as Euler who was a self-taught prodigy close to Bernoulli by the vogue and understanding of life.

Observe that Wolff was the trendsetter in mathematical education of the beginning of the eighteenth century. After Leibniz's refusal to transfer to Saint Petersburg for organizing the Academy, Peter considered Wolff as its possible leader. Wolff's treatise *Der Anfangsgründe aller mathematischen Wissenshaft* was published in four parts in 1710, abridged later for a wider readership, and reprinted many times (cp. [15, p. 23]).

Explicating his pedagogical principles, Wolff wrote (cp. [13]):[1]

> In my lectures I paid most attention to the three aspects:
> (1) I never used any word that was left unexplained before in order to avoid ambiguity and logical gaps;
> (2) I never used any theorem I had not proved before;
> (3) I always connected theorems and definitions with one another to make a continual logical chain.
> It is universally known that these rules are followed in mathematics. If we compare the mathematical method of education with the approach of logic, we can see that the mathematical method of education is nothing other than the exact application of the rules of inference. Therefore it is immaterial whether we use

---

[1]Unfortunately, the next quotation is not a direct translation of Wolff's original.

the mathematical method of education or the rules of inference provided that the latter are true. Inasmuch I have demonstrated that the mathematical reasoning reflects the natural reasoning, and the logical reasoning is just a definitely improved form of the natural reasoning, I have any right to declare that my method of education follows the natural mode of reasoning.

Hegel was rather sceptical about the pedagogical style of Wolff and remarked (cp. [14, p. 363]):

Wolff on the one hand started upon a large range of investigation, and one quite indefinite in character, and on the other, held to a strictly methodical manner with regard to propositions and their proofs. The method is really similar to that of Spinoza, only it is more wooden and lifeless than his.

The ideas of Wolff in education were well accepted by Lomonosov, since Wolff and he were connected with the warm feeling of mutual respect. Wolff's mathematical method was a basis of Lomonosov's scientific articles during many years of his creativity. It should be observed that , unlike Wolff who had excellent mathematical training, Lomonosov was not sufficiently well acquainted even with Euclid's *Elements*, and he never possessed a working knowledge of differential and integral calculus.

We must emphasize that Lomonosov never met Euler (mentioning this in his famous talk "Lomonosov and World Science" [16], Kapitsa made an exquisite circumlocution "of course we cannot exclude the possibility of Lomonosov presence at the public lecture of Euler that he delivered before his departure to Germany"). These circumstances explain to us we can hardly find any practical applications of mathematics in the papers of Lomonosov and why some of his thoughts about the nature of mathematical knowledge are naive and incorrect.

For instance, in his great paper *Meditationes de Caloris et Frigoris Causa Auctore Michaele Lomonosow* propounding foundations of the molecular-kinetic theory of heat (cp. [17, Cp. 1]), Lomonosov wrote [18, c. 24]:

Nulla demonstrandi methodus certior est ea mathematicorum, qui deductas a priori propositiones exemplis vel examine institute a posteriore confirmare solent.

There is no more certain method of demonstration than the method of mathematicians who corroborate the *a priori* deduced propositions by example and *a posteriori* examination.

It is worth emphasizing that from this formally wrong thesis about the nature of mathematical proof, Lomonosov deduced the remarkable and undoubtedly true conclusion:

Idcirco nostram theoriam ulterius prosecuturi, ad exemplum eorum phaenomena praecipua, quae circa ignem et calorem observantur, explicando assertum § 11 verissimum esse confirmabimus.

Therefore, to further develop our theory we use the lead of mathematicians and explain the most important phenomena that are observed for fire and heat, thus corroborating the full validity of the assertion we made in §11.

In actuality, Lomonosov discussed in this excerpt the technology of mathematical modeling which differs drastically from any mathematical formalism as such.

The attitude of Lomonosov to monads deserves a slightly thorough examination. Developing the atomistic ideas of corpuscular physics in his papers of 1743 and 1744,

i. e. *Tentamen Theoriae De Particulis Insensibiubus Corporum Deque Causis Qualitatum Particularium in Genere, De Cohaesione Et Situ Monadum Physicarum,* and it De Particulis Physicis Insensibilibus Corpora Naturalia Constituentibus, in Quibus Qualitatum Particularium Ratio Suffic1ens Continetur ([19, pp. 169–235, 265–314]) as well as in his extensive correspondence, Lomonosov sparingly use the concept of monad, especially distinguishing *monades physicae.* The physical monads of Lomonosov are closer to the conception of atoms rather than mathematical monads or Leibnizian ideal monads. Long-term personal contemplations over the structure of the matter led Lomonosov (e.g., see [20]). This is reflected in the choice of Latin scientific terms of the later papers of Lomonosov (cp. [21]).

In February of 1754 Lomonosov wrote to Euler [22, pp. 501-502]:

> Fateor me idcirco potius ilia praeteriisse, ne magnorum virorum scripta invadendo sui ostentator viderer potius, quam veritatis scrutator. Haec ipsa ratio jam longo tempore prohibet, quominus meditationes meas de monadibus erudito orbi proponam discutiendas. Quamvis enim mysticam fere illam doctrinam funditus everti argumentis meis debere confidam; viri tamen, cujus erga me officia oblivisci non possum, senectutem aegritudine animi affigere vereor; alias crabrones monadicos per totam Germaniam irritare non perhorrescerem.

> I confess that I avoided all this also by the reason that I did not want to look as a bragger attacking the writhing of the great scholars rather the a man pursuing truth. The same reason has been preventing for a long time to submit for consideration of the scientific council my views of monads. Although I am absolutely convinced that this mystical teaching must be completely destroyed by my arguments, I am afraid to spoil the elder years of the scholar whose benefactions to me I could not forget; otherwise I would not be scared to tease hornet-monadists throughout the whole of Germany.

It is worth observing that Lomonosov had in mind not the idea of Leibniz himself but rather the exposition of monadism in the writings of Wolff and his numerous descendants. This day we know Wolff's letter to Ernst Christoph von Manteuffel as of May 11, 1746 which shows that Wolff considered his metaphysics as different from that of Leibniz (cp. [23]).

Let us make a mental "physicalistic" experiment and aim a strong microscope at a region about a point at a mathematical line. We will see in the eyepiece a blurred and dispersed cloud with unclear frontiers which is a visualization of the point under investigation. Under greater magnification, the portion of the "point-monad" we are looking at will enlarge, revealing extra details whereas disappearing partially from sight. However, we are still inspecting the same standard real number which you might prefer to percept as described by this process of "studying the microstructure of a physical straight line." Visualizing a point by microscope reveals its monadic essence. Lomonosov and even Leibniz could reason likewise or approximately so. In any case, the view of the monad of a standard real number as the collection of all infinitely close points is generally adopted in the contemporary infinitesimal analysis resurrected under the name of *nonstandard analysis* in the works by Abraham Robinson in 1961 (cp. [24], [25])..

More than two dozens of decades elapsed from the death of Mikhaĭlo Lomonosov, but his creative contribution still inspires thought in connection with the most topical and brand-new areas of mathematics and natural sciences. His enviable fate gives a supreme example for drafting life.

References

[1] Henri V. A.. "The Role of Leibnitz in the Establishment of Scientific Schools in Russia,"
Phys. Usp., **42**, 1223–1225 (1999).

[2] Hellenbroich E., "G. W. Leibniz and the Ecumenical Alliance of All Eurasia," *Fidelio*, **5**:3
(1996).

[3] Smith J., "Leibniz Manuscript Transcriptions,"
\protect\vrule width0pt\protect\href{http://www.jehsmith.com/philosophy/2009/01/peter-the-greats-decree-appoint
-leibniz-to-the-russian-justizrat.html

[4] Cracraft J. *The Petrine Revolution in Russian Culture.* Cambridge and London: The Belknap
Press of Harvard University Press, 2004

[5] Vavilov S. I., *Isaak Newton.* Moscow and Leningrad: AN SSSR, 1945.

[6] Ekeland I., *The Best of All Possible Worlds. Mathematics and Destiny.* Chicago and London:
The University of Chicago Press, 2006.

[7] Keynes J. M., "Newton, the Man,"
\protect\vrule width0pt\protect\href{http://www-history.mcs.st-and.ac.uk/Extras/Keynes_Newton.html}{http://www-

[8] Child J. M., *The Early Mathematical Manuscripts of Leibniz.* Mineola: Dover Publications,
2005 (First Published in 1920 by The Open Court Publishing Company, Chicago).

[9] Leibniz G. W., *Collected Works. Vol. 1.* Moscow: Mysl, 413–428, 1982.

[10] Euclid, *Elements, In Three Volumes.* Moscow and Leningrad: Gostekhizdat, 1949.

[11] Sextus Empiricus, *Collected Works. Vol. 1.* Moscow: Mysl , 1976.

[12] Frängsmyr T., Heilbron J. L., and Rider R.E. (Eds.), *The Quantifying Spirit in the 18th
Century.* Berkeley etc.: University Of California Press (1990).

[13] Rogers L., "Is the Historical Reconstruction of Mathematical Knowledge Possible?"   In:
Histoire et Epistemologie dans l'Education Mmathematique. IREM de Montpellier, 1995,
pp. 105–114.

[14] Hegel, Selected Works. Vol. 11. Moscow and Leningrad: Sotséguiz, 1935.

[15] Yushrevich A. P. (Ed.), *History of Mathematics from the Most Ancient Times to the Begin-
ning of the Nineteenth Century.* Vol. 3. Moscow: Nauka, 1972.

[16] Kapitsa P. L., "Lomonosov and World Science," *Sov. Phys. Usp.*, **8**, 720–728 (1966).

[17] Kutateladze S. S. and Tsukerman R. V., *Development of the Theory of Heat in the Works
of Russian Scientists.* Moscow and Leningrad: Gosenergoizdat, 1949.

[18] Lomonosov M. V., *Complete Works. Vol. 2.* Moscow and Leningrad: AN SSSR, 1951.

[19] Lomonosov M. V., *Complete Works. Vol. 1.* Moscow and Leningrad: AN SSSR, 1950.

[20] Gareth Jones W., "Russia's Eighteenth-Century Enlightenment," In: Leatherbarrow W. and
Offord D. (Eds.) *A History of Russian Thought.* Cambridge: Cambridge University Press,
2010, pp. 73–94

[21] Smirnova A. S. and Volkov S. Sv., "Atomus, Monas and Corpusculum in Lomonosov's Arti-
cles on Natural Sciences," In: *Indo-European Lexicology and Classical Philology—XI.* Saint
Petersburg: Nestor–Istoria, 2007. pp. 278–284.

[22] Lomonosov M. V., *Complete Works. Vol. 10.* Moscow and Leningrad: AN SSSR, 1957.

[23] Lenders W., "The analytic Logic of G. W. Leibniz and Chr. Wolff: A Problem in Kant
Research," *Synthese*, **23** 147–153 (1971).

[24] Robinson A., *Non-Standard Analysis. With a foreword by Wilhelmus A. J. Luxemburg.*
Princeton: Princeton University Press, 1996 (A Paperback Reprint of the 2nd edition).

[25] Gordon E. I., Kusraev A. G., and Kutateladze S. S., *Infinitesimal Analysis.* Dordrecht:
Kluwer Academic Publishers, 2002.

Sobolev Institute of Mathematics
4 Koptyug Avenue
Novosibirsk, 630090
RUSSIA
E-mail address: sskut@math.nsc.ru